Private Information Discovery, Value-Risk Tradeoffs, and Public Disclosure

Hao Xue

October 16, 2019

1I thank Qi Chen, Wayne Guay, Jonathan Glover, Xu Jiang, Richard Lambert, Rahul Vashisht, and Ronghuo Zheng for their constructive comments. The author is at Fuqua School of Business, Duke University.
Abstract

This paper models a manager’s value-risk tradeoff in operating a firm when managerial incentives are tied to the stock price in capital market. We examine how the manager’s actions and market information environment would change as investors’ information acquisition cost is reduced. Consistent with empirical evidence, we show that a lower information acquisition cost motivates more investors to acquire information, improves price informativeness, and increases market depth. However, these “benefits” occurs at a cost of inducing the manager to pursue less risky (hence, lower value) projects to the extent that the change not only lowers the firm’s expected value but also can reduce trading profit investors earn in the capital market. The results suggest that a decrease in investor’s information acquisition cost is like a prisoner’s dilemma: each individual investor is better off ex post, collectively, however, investors can be worse-off ex ante. We show that public disclosure is an efficient way to mitigate the dilemma and derive a unique interior disclosure quality that maximizes the expected firm value.

JEL Classifications: D82; G14; M41
1 Introduction

Price in the capital market disciplines the manager's choices, and the effectiveness of the disciplinary role depends on price informativeness (e.g., Holmström and Tirole, 1993; Edmans and Manso, 2011). Because market price aggregates investors' private information, a key determinant of the price informativeness, and, hence its disciplinary role, is how costly it is for investors to acquire value-relevant private information. The general belief is that firms with lower information acquisition cost are associated with more informative price. Recent technological innovations has lowered the information acquisition cost so much that it becomes cost efficient for asset management companies to collect real-time satellite images and consumer transaction data for valuation purposes. Empirical evidence suggests that the introduction of these technological innovations increases price informativeness and market liquidity (e.g., Blankespoor et al., 2014). Zhu (2019) documents an increased price informativeness and a stronger disciplinary role of price over managers' actions when the decreased information acquisition cost made “big data” available to investors.

In this paper, we explicit model individual investors' information acquisition and examine the disciplinary role of market price for managerial decisions as one lowers investors' private information acquisition cost. Consistent with empirical evidence and general beliefs, we show that lowering investors’ information acquisition cost indeed leads to more investors to be informed, improves price informativeness, and improve market liquidity. However, these “benefits” occurs at a cost of inducing managers to pursue less risky (hence, lower return) projects to the extent that the change not only lowers the firm expected value but also often reduces investors’ ex ante payoff. We also investigate how the firm can help improve firm value and investors welfare by change its disclosure policy.

We model a firm whose share is traded in a capital market, a continuum of risk-averse investors, and a manager who operates the firm. The risk-averse manager (he) exerts unobservable efforts along two dimensions. On one hand, the manager exert a mean-improving
effort that increases the distribution of the firm’s cash flow in the sense of first-order stochastic dominance. On the other hand, the manager can also engage in costly risk management activities aimed to reduce the risk of the underlying cash flow in the sense of second-order stochastic dominance. The manager’s risk management activities can be (ex ante) screening and selection effort before the project is taken and/or (ex post) hedging and monitoring activities during the tenure of the project. The manager sells his share in the capital market in which the price is determined endogenously. As in Grossman and Stiglitz (1980), each of the continuum of investors chooses whether acquire a private information at a cost. We study how a reduction in the information acquisition cost affects the manager’s and investors’ behaviors as well as properties of the capital market.

We start by analyzing a benchmark in which the manager cannot engage in risk management activities and, hence, the risk of the underlying cash flow is exogenous. This single-task benchmark is consistent with most of the existing models that study the role of using equity to provide managerial incentives. We show that more investors will choose to become informed if the cost of doing so is lower. Consistent with prior literature, we show that inducing a higher fraction of informed investors makes the price more informative, which in turn, motivates the manager to exert more effort. Intuitively, as more investors choose to be informed, equilibrium pricing is more responsive to the true effort as opposed to the conjectured effort level that the manager cannot change. This increases the manager’s perceived marginal benefit of effort and hence his equilibrium effort choice.

A decrease in the information acquisition cost has a much subtler effect once we account for the fact that the manager can also influence the riskiness of the project the firm takes. On the one hand, we continue to show that lowering investors’ information acquisition cost will lead to more investors to be informed, improve price informativeness, and an increase in market liquidity. On the other hand, we show that these observed “benefits” occur at a cost of inducing managers to pursue lower return projects to the extent that it lowers the firm value. In fact, if the mean-improving effort and the risk-reducing activities are assumed to
be equally costly for the manager, reducing investor’s private information acquisition cost disproportionally benefits the manager’s risk-reducing effort to the extent that the manager will *always* shift his attentional away from his mean-improving effort and into the risk-reducing effort.

In addition to motivating manager to take on low-risk and low-return projects, facilitating investors’ private information discovery also affects investors expected trading profit by changing the very distribution of the underlying risky asset that they will trade on. We derive investors expected trading profit (net of the information acquisition cost) in closed-form. Given that the investors are risk averse, there is potential for the investors to benefit from firm investing in projects with more predictable (i.e., lower risk) payoffs. However, we show that investors will not benefit from having safer projects unless the percentage of informed investors are sufficiently high to start with, which occurs only if the cost of information acquisition is already sufficiently low. This result is surprising to us. Contrary to what casual intuition would suggest, facilitating investors private information discovery will not benefit investors unless the information environment is already transparent enough. For companies that have relative opaque information environment (i.e., high information acquisition cost and, hence, only a small fraction of informed investors), encouraging private information discovery makes investors strictly worse off. The result is analogous to a prisoner’s dilemma. It is true that lowering information acquisition cost makes each individual investor better off *ex post*. Collectively, however, the lower cost incentivizes too many investors to acquire information, which not only pressures the manager to take on lower return projects but also irons out ex-ante price variations and, hence, destroys the expected trading profit.

We also investigate the role of public disclosure in mitigating the “prisoner’s dilemma” shown above. The result shows that providing public disclosure can be an effective way to overcome the manager’s distorted incentives to chase low value-and-risk projects. We derive a unique, interior disclosure quality that maximizes the expected firm value. As we improve disclosure quality towards the optimal disclosure quality, more precise disclosure not
only increases the expected firm value but also improves investors’ expected trading profit. Improving disclosure quality beyond the optimal level, however, generates an endogenous cost of reducing the firm value. This is because the manager will stop exerting any risk-reducing effort when investors’ posterior assessment of the firm’s underlying risk is precise enough upon observing the disclosure, which occurs when the disclosure is sufficiently precise. In this case, the manager’s incentive problem degenerates to a single-dimension task in which he focuses exclusively on improving the expected firm value. While improving disclosure quality directly reveals signals about the manager’s effort choice to the investors, it indirectly crowds out the percentage on informed investors and, hence, the information contained in their trades. We show that, when the manager’s effort is single-dimensional, the endogenous cost of higher disclosure quality in crowding out investors’ private information discovery is the dominant effect.

The paper is related to the literature that studies the disciplinary role of market price in providing managerial incentives. Existing models typically assume that the manager’s action is single dimensional and that a higher action increases the firm value in the stochastic sense (e.g., Holmström and Tirole, 1993; Edmans and Manso, 2011; Xue and Zheng, 2019). The novel part of our model is to allow the manager’s actions to be multi-dimensional: the manager’s unobservable actions affect both the mean and the variance of the underlying cash flow. The multi-dimensional feature allows us to capture the value-risk tradeoff intrinsic to many investment decisions that existing models are inadequate to capture. Tsui (2018) document evidence consistent with firms facing value-risk tradeoff in investment choices.

Lambert (1986) studies the manager’s incentives to adopt risky project in a contracting setting. In his model, a manager exerts unobservable effort to generate information about the profitability of a risky project and, conditional the information his effort generates, chooses between the risky project and a fallback safe project. He shows that the risk-averse manager and the risk-neutral principal will not always agree regarding which project is better. This is because the principal does not observe either the manager’s effort choice nor
the specific information his effort generates. Our model differs in two major ways. First, whereas the distribution of the risky project is taken as exogenous given in his model, the manager in our model directly chooses the riskiness of the firm’s cash flow (as well as its mean). Endogenizing the riskiness of the project helps to capture the value-risk tradeoff the manager faces. Second, Lambert (1986) studies the role of incentive contract in distorting the manager’s effort provision, while the pressure comes from investors’ private information acquisition and trading in the capital market.\footnote{An important difference is that manager’s actions and investors’ information acquisition are determined simultaneously. In contrast, incentive contract is determined publicly before the manager chooses his effort in Lambert (1986).}

This paper is also related to the broad literature that studies how capital market pressure can distort managers’ investment choices. Bebchuk and Stole (1993) show that information frictions of different nature can lead the firm to either underinvest or overinvest. Kanodia and Lee (1998) show that periodic performance reports play an important role in balancing the manager’s investment incentives. Gigler et al. (2014) study the cost-benefit tradeoff of increasing the frequency of financial reporting. They show that when the firm’s project choice is unobservable, increasing reporting frequency increases the price pressure on the firm and induces managerial myopia. Empirical evidence suggests that firms’ investment decision is more myopia when facing more frequent financial reporting and when the institutional investors are short-term focused (e.g., Kraft et al., 2017; Agarwal et al., 2017). Existing studies focus on myopia, which is an inter-temporal argument by nature, and show that incentive to boost current period price induces the managers to engage activities that borrow further value at an unfavorable rate. The manager in our model is risk averse and, hence, also concerned about the volatility of the price (see Amihud and Lev, 1981; Smith and Stulz, 1985 for empirical evidence that managers take actions to reduce investment and price volatilities). Our result complements the myopia argument with the value-risk tradeoff.

We organize the paper as follows. Section 2 introduces the model setup. Section 3 studies a benchmark in which the manager cannot affect the riskiness off the firm. Section
4 allows the manager also to engage in unobservable risk management activities that lowers the risk of the underlying project. We shows that, unlike in the benchmark model, lowering investors private information acquisition cost motivates the manager to chase projects with lower risk at a cost of lower value. Section 5 studies investors’ welfare and derive conditions under which motivating low value-and-risk project is beneficial for the investors. Section 6 investigates how public disclosures can help improve the firm value as well as investors’ ex ante payoffs. Section 7 concludes.

2 Model Setup

The model consists of a firm, a manager, and a continuum of investors. The manager operates the firm and takes two unobservable actions $a_1$ and $a_2$ that affect the mean and the variance of firm value, respectively. Denote by $v$ the firm value, we assume that

$$v = \theta + e,$$  \hspace{1cm} (1)

where $\theta \sim N(a_1, \sigma_\theta^2)$ is normally distributed: its mean increases in the manager’s productive effort $a_1$ and its variance $\sigma_\theta^2$ decreases in the manager’s risk-reduction effort $a_2$ as follows:

$$\sigma_\theta^2 = \Sigma - a_2.$$  \hspace{1cm} (2)

Put differently, the manager’s productive effort $a_1$ increases the firm’s value in the sense of first-order stochastic dominance; while his risk reducing effort $a_2$ reduces the “prior” risk of the firm in the sense of second-order stochastic dominance. The stochastic term in (1) $e \sim N(0, \sigma_e^2)$ is normally distributed with an exogenous variance $\sigma_e^2$. The manager’s personal cost of exerting effort $a_1$ and $a_2$ is

$$C(a_1, a_2) = \frac{(a_1 + a_2)^2}{2}.$$  \hspace{1cm} (3)
Given the manager’s unobservable actions $a_1$ and $a_2$, the firm is traded in a competitive market in which the price $p$ is determined. The manager chooses his actions $a_1$ and $a_2$ privately to maximize his payoff:\footnote{The mean-variance preference can be derived by assuming that the manager has a negative exponential utility: $U^M = -\exp \left[ -\rho (p - C(a_1, a_2)) \right]$. That is, the manager maximizes the cash generated from selling his share, which is normalized to 1, at price $p$, net of the cost of effort. It is well-known that the manager maximizes his certainty equivalence $E[p] - \rho \text{var}(p) - C(a_1, a_2)$, which is qualitatively similar to (4).}

$$U^M = E[p] - \rho_M \text{var}(p) - C(a_1, a_2),$$  \hspace{1cm} (4)

where $\rho_M$ is the manager’s risk aversion.

The market-clearing price $p$ shown in (4) is determined in a competitive market à la Grossman and Stiglitz (1980). There is a continuum of investors $i \in [0, 1]$ and a risk-free asset that serves as the numeraire. Each investor is endowed with $w_0$ units of the risk-free asset. The capital market is populated by two types of investors: rational investors and noisy traders. Rational investors have constant absolute risk averse (CARA) utility functions with a common risk-aversion parameter $\rho > 0$. Before submitting their trade, each of the investor chooses whether to pay an information acquisition fee $F$ to learn the realization of $\theta$ perfectly. We call those who pay to learn $\theta$ informed investors and those who choose not to learn $\theta$ uninformed investors. Uninformed investor only observe the market price $p$, while informed investors, by paying $F$, also learns $\theta$ (in addition to $p$).\footnote{As in Grossman and Stiglitz (1980), $e$ in (1) is unlearnable to anyone in the model. Model like Grossman and Stiglitz (1980) needs an un-learnable component because, otherwise, informed investors learns the firm value perfectly and would have an unlimited demand whenever the price is away from the firm value.} Denote $\lambda$ be the fraction of investors who choose to become informed in equilibrium. Noise traders provide liquidities in the sense that they supply $\epsilon$ units of the firm’s share per capita to the capital market, and we assume $\epsilon \sim N(0, \sigma_\epsilon^2)$. Price $p$ is determined to clear the market, that is, to equal the aggregated demand and the random supply $\epsilon$. Figure 1 summarizes the sequence of events.

What distinguishes our model from existing studies on investors’ information acquisition is the integration of the manager’s multi-task unobservable actions. That is, while the
underlying distribution of the firm value has been taken exogenously given in earlier studies, the distribution itself – both the mean and the variance – is a function of the manager’s multi-dimensional actions. Because the manager’s actions are unobservable, the investors need to form conjectures $\hat{a}_1$ and $\hat{a}_2$ in deciding whether to become informed and in submitting their trades. The (anticipated) investors’ information acquisition, in turn, will affect the manager’s allocation of efforts between his two tasks, and hence the distribution of the firm value. In equilibrium, all conjectures are consistent with their actual value.

3 The Single-Task Benchmark

We first study a benchmark in which the manager can only improve the mean of the firm’s cash flow and cannot influence its riskiness. That is, the risk-reduction effort $a_2$ is set to be zero by assumption. The sequence of events follows the main model described above: the manager chooses a single unobservable effort $a_1$ at $t = 0$, $\lambda$ fraction of investors pay $F$ to become informed at $t = 1$, both informed and uninformed investors trade and market-clearing price $p$ is determined at $t = 2$.

Because investors do not observe the manager’s effort choice $a_1$, they use the conjectured
\( \hat{a}_1 \) in making their trading decisions. We solve the equilibrium using backward induction. First, we solve for the subsequent trading game at \( t = 1 \), taking investors’ conjecture \( \hat{a}_1 \) about the entrepreneur’s unobservable effort at \( t = 0 \) as given. In the second step, we take the subgame pricing function as given and solve for the manager’s effort choices. The equilibrium is then fully characterized by requiring conjectured effort choices to be correct in equilibrium.

Once we have taken the manager’s conjectured effort \( \hat{a}_1 \) as given, the distribution of the firm value is “known” to investors. In this case, solving the subsequent trading game is standard following Grossman and Stiglitz (1980). We know that, fixing the market’s conjecture \( \hat{a}_1 \), there exists a unique linear pricing function as follows that clears the market

\[
p(\hat{a}_1) = A + B \times \theta - C \times \epsilon,
\]

where the coefficients \( A, B, \) and \( C \) are non-negative constant specified later.

At \( t = 0 \), the manager takes as given the pricing function (5) and chooses his effort \( a_1 \) to maximize his utility. That is,

\[
a_1^* \in \arg \max_{a_1} E[p|\hat{a}_1] - \rho Var(p|\hat{a}_1) - \frac{a_1^2}{2}.
\]

Differentiating the payoff function with respect to \( a_1 \), we obtain the following first-order condition (FOC) for the optimal \( a_1^* \)

\[
a_1^* = B.
\]
and, hence, is only partially responsive to the manager's true effort choice $a_1$. This can be seen by calculating the expected price $E(p)$ from the manager's perspective when choosing his effort $a_1$:

$$E[p|a_1] = A\hat{a}_1 + Ba_1.$$  

The manager takes investors’ conjecture $\hat{a}_1$ as given and cannot influence it. It is clear that a higher effort $a_1$ increases the expected price at a rate $B < 1$. In other words, $B$ captures the manager’s perceived marginal benefit of effort.

To complete the characterization of the equilibrium, we require that the conjecture is correct in equilibrium. That is, the conjectured effort equals the actual one in equilibrium (i.e., $\hat{a}_1 = a_1^*$). Proposition 1 below summarizes the equilibrium and its comparative statics.

**Lemma 1** There exists a unique linear pricing function that clears the market:

$$p = A + B \times \theta - C \times \epsilon,$$

where the non-negative coefficients $A = \frac{(1-\lambda)\tau_\theta}{\lambda \tau_e + \tau_\theta + \tau_p} a_1^*$, $B = \frac{\tau_p + \lambda (\tau_e + \tau_\theta)}{\lambda \tau_e + \tau_\theta + \tau_p}$, and $C = \frac{\rho(\tau_e + \tau_\theta + \tau_p/\lambda)}{\tau_e (\lambda \tau_e + \tau_\theta + \tau_p)}$ are functions of the fraction $\lambda$ of informed investors, and $\tau_p = (\lambda \tau_e/\rho)^2 \tau_e$ is the precision of price used as a signal of $\theta$. In equilibrium, $\lambda = 1$ for $F < \hat{F}$ and $\lambda = 0$ for $F > \bar{F}$. For $F \in [\bar{F}, \hat{F}]$, $\lambda$ is uniquely determined by the following equation

$$\exp(\rho \ast F) = \sqrt{\frac{\lambda^2 \Sigma + \rho^2 \sigma^2 \sigma_e^2 (\sigma_e^2 + \Sigma)}{\lambda^2 \Sigma + \rho^2 \sigma^4 \sigma_e^2}}.$$  

The manager’s unobservable effort is $a_1^* = B$ in equilibrium.

**Proof.** All proofs are in the Appendix. ■

Not surprisingly, no one will choose to be informed if the information acquisition cost is sufficiently high, i.e., $F > \bar{F}$; while all investors will be informed if the cost is sufficiently low, i.e., $F < \hat{F}$. (We specify the two thresholds $\bar{F}$ and $\hat{F}$ in the appendix). In the remainder of
the paper, we maintain the assumption $F \in [F, \tilde{F}]$, which allows us to focus on the interior equilibrium (i.e., $0 < \lambda < 1$).

The following result follows directly from Lemma 1, and it summarizes the effect of a lower information acquisition cost on equilibrium outcomes.

**Corollary 1 (Comparative statics)** As investors’ information acquisition cost $F$ is lower,

(i) more investors become informed, i.e., higher $\lambda$;

(ii) the manager exerts higher effort, i.e., higher $a^*_1$.

Part (i) of the proposition is intuitive. More investors choose to become informed if the cost of doing so is lower. Part (ii) highlights the disciplinary role of the equilibrium informativeness. The key is to note that as more investors choose to be informed (i.e., higher $\lambda$), equilibrium pricing is more responsive to the manager’s true effort $a$ as opposed to the conjectured $\hat{a}$. As a result, a higher $\lambda$ results in a higher pricing coefficient $B$ in (5), which increases the manager’s perceived marginal benefit of effort and hence his equilibrium effort choice.

### 4 Equilibrium When Firm Can Manage Its Risk

Manager’s job is complex and often multi-dimensional. In addition to being able to exert costly effort to improve the expected firm value, the manager can also engage in costly risk management activities that reduce the risk firm is facing. For example, such risk management activities can be either ex ante screening and project selection before the project is taken and/or ex post risk hedging and monitoring activities during the tenure of the project.

Incorporating the manager’s risk management effort $a_2$ complicates the equilibrium substantially. Inspecting Lemma 1, we notice that all equilibrium variables (such as pricing coefficients and investors’ information acquisition) are functions of the variance $\Sigma$ of the
firm value. Once we allow for the manager’s risk management activity \( a_2 \), the “prior” variance that was taken exogenously in Lemma 1 is replaced with \( \Sigma - a_2 \) and, hence, becomes an endogenous variable. Furthermore, since the manager’s action \( a_2 \) is unobservable, investors have to form a conjectured \( \hat{a}_2 \) and used the conjectured prior variance in deciding whether to become informed and how much to trade.

**Definition:** An equilibrium is a collection of the manager’s actions \( a_1, a_2 \), investors’ information acquisition choice \( \lambda \), trading decisions, and a pricing function \( p \) such that:

1. Given the conjectured \( (\hat{a}_1, \hat{a}_2) \) and the fraction \( \lambda \) of investors who are informed, an investor is indifferent between paying \( F \) to become informed and staying uninformed.

2. Both informed and uninformed investors’ trade to maximize their CARA utilities.

3. Given \( \lambda, \hat{a}_1, \hat{a}_2 \) (and hence the pricing function), the manager chooses \( a_1^{BR} \) and \( a_2^{BR} \) to maximize his utility.

4. Rational expectation: \( \hat{a}_1 = a_1^{BR}, \hat{a}_2 = a_2^{BR} \).

5. Market clears under the linear pricing function \( p \).

We solve the equilibrium using backward induction as we did in the single-task benchmark. We first take the market conjectured efforts \( \hat{a}_1 \) and \( \hat{a}_2 \) as given and characterize the unique linear pricing function that clears the market. It is easy to see that, given conjectures \( \hat{a}_1 \) and \( \hat{a}_2 \), the linear pricing function is essentially the same as (7) in Lemma 1, whereas the only difference is to replace the prior variance \( \Sigma \) with \( \Sigma - \hat{a}_2 \). We rewrite the pricing function as following to emphasize its reliance on the conjectured efforts:

\[
p = p(\hat{a}_1, \hat{a}_2) = A(\hat{a}_1, \hat{a}_2) + B(\hat{a}_1, \hat{a}_2) \times \theta - C(\hat{a}_1, \hat{a}_2) \times \epsilon. \tag{8}
\]

Given the pricing function above, we examine the manager’s effort choices at \( t = 0 \). That is, the manager’s best-response \( a_1^{BR} \) and \( a_2^{BR} \) as a function of conjectured \( (\hat{a}_1, \hat{a}_2) \) and, hence,
the pricing function above. We know from the optimality that the manager’s best responses $a_1^{BR}$ and $a_2^{BR}$ must satisfy

$$a_1^{BR}, a_2^{BR} \in \arg\max_{a_1, a_2} E[p|\hat{a}_1, \hat{a}_2] - \rho_M \text{var}(p|\hat{a}_1, \hat{a}_2) - \frac{(a_1 + a_2)^2}{2}. $$

One can derive the first-order conditions for $a_1^{BR}$ as

$$B(\hat{a}_1, \hat{a}_2) = a_1 + a_2, \quad (\text{FOCa}_1)$$

and, similarly, the first-order condition for $a_2^{BR}$ is

$$\rho_M B^2(\hat{a}_1, \hat{a}_2) = a_1 + a_2. \quad (\text{FOCa}_2)$$

We have explained in the benchmark why the pricing coefficient $B$ captures the marginal benefit of the manager’s mean-improving effort $a_1$. The same intuition underlines the left-hand side of (FOCa$_1$). Inspecting the left-hand side of (FOCa$_2$), one can see that a higher pricing coefficient $B$ also increases the manager’s marginal benefit of risk-reducing effort $a_2$. To see the intuition, note that the variance of the price from the manager’s point of view at $t = 0$ is

$$\text{Var}(p) = B^2(\hat{a}_1, \hat{a}_2) \times (\Sigma - a_2 + \sigma_e^2) + C^2(\hat{a}_1, \hat{a}_2) \times \sigma_e^2. $$

All else equal, price becomes more volatile for higher $B^2(\hat{a}_1, \hat{a}_2)$. Intuitively, a higher $B$ means that the price is tied more closely to its fundamental value $\theta$, making the price more revealing ex-post. However, linking the price closer to the fundamental makes the price more volatile from the ex-ante point of view. The finding is consistent with the traditional wisdom that better information reduces uncertainty ex post but increases the uncertainty ex ante (e.g., Christensen et al., 2010; Dutta and Nezlobin, 2017; Gao, 2010). A higher price volatility lowers the manager’s payoff because he is risk averse. The marginal benefit of $a_2$ in
increasing the manager’s payoff is \( \frac{dU^M}{da_2} = -\frac{d\rho M \text{var}(p)}{da_2} = \rho M B^2(\hat{a}_1, \hat{a}_2) \), which is the left-hand side of (FOCa_2).

We determine the equilibrium by imposing rational expectation conditions and summarize the equilibrium in the proposition below.

**Proposition 1** The equilibrium \( a_1, a_2 \) and \( \lambda \) are characterized by the following system of equations:

\[
\begin{align*}
\text{FOC}_{a_1}(a_1, \hat{a}_2) &= 0 \quad (9) \\
\text{FOC}_{a_2}(a_2, \hat{a}_2) &= 0 \quad (10) \\
&
\end{align*}
\]

\[
\exp(\rho \ast F) - \sqrt{\frac{\lambda^2(\Sigma - \hat{a}_2) + \rho^2 \sigma_e^2 \sigma^2 + \Sigma - \hat{a}_2}{\lambda^2(\Sigma - \hat{a}_2) + \rho^2 \sigma_e^2 \sigma^2}} = 0 \quad (11)
\]

\[
a_1 = \hat{a}_1 \quad (12)
\]

\[
a_2 = \hat{a}_2. \quad (13)
\]

*The unique linear pricing function is determined by substituting \( a_1, a_2 \) and \( \lambda \) derived above back to Lemma 1 and replacing \( \Sigma \) with \( \Sigma - a_2 \).*

Table 1 illustrates the equilibrium actions via two numerical examples in which investor and the manager’s risk aversion \( \rho = \rho_M = 1.5 \), and variance parameters are \( \Sigma = 3, \sigma_e^2 = 2.5 \), and \( \sigma^2 = 3 \). In Table 1, we present equilibrium actions \((\lambda, a_1, a_2)\) under two information acquisition cost \( F \).

<table>
<thead>
<tr>
<th>( F )</th>
<th>( \lambda )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.48</td>
<td>0.5</td>
<td>0.17</td>
</tr>
<tr>
<td>0.23</td>
<td>0.50</td>
<td>0.19</td>
<td>0.48</td>
</tr>
</tbody>
</table>

*Table 1: Numerical examples of the equilibrium (for \( \Sigma = 3, \sigma_e^2 = 2.5 \), and \( \sigma^2 = 3 \).)*

As is clear from the example above, when the investors’ information acquisition cost \( F \)
is lower, more investors choose to become informed: $\lambda$ increases from 48% to 50%. This is intuitive and is the same as what we have shown in the single-task benchmark.

It is interesting to examine Table 1 regarding how the manager’s equilibrium actions are affected by a decrease in information acquisition cost. Contrary to the single-task benchmark, a lower $F$ actually reduces the manager’s mean-improving effort $a_1$ (from 0.5 to 0.19 in Figure 1). In contrast, the anticipation of more informed investors motivates the manager to allocate more attention to the risk-reducing effort $a_2$, which is increased from 0.17 to 0.48. In other words, reducing investors’ information acquisition cost $F$ changes the firm’s informational environment in the way that incentivizes the manager to shift their effort away from improving the mean return of the firm into managing the underlying risk of the firm.

The manager’s de-emphasis of the firm’s average return is a novel aspect of the paper. Stock price has a well-understood role of providing managerial incentives (e.g., Holmström and Tirole, 1993; Edmans and Manso, 2011). Nevertheless, the example above highlights an endogenous limit of using market price to provide managerial incentives when the manager’s job is multi-dimensional. In particular, when a risk-averse manager needs to allocate his effort (or limited time) between improving average return of the firm and reducing its risk, incentives tied to capital market price tends to disproportionally promote the manager’s risk-reduction incentive at the cost of lowering the firm’s average profitability.

Features illustrated in the numerical example can be proved analytically. In particular, we apply the implicit function theorem to the system equations used to determine the manager’s actions $a_1, a_2$ and investors’ choice $\lambda$. Let $[g_1(a_1, a_2, \lambda; F) = 0, g_2(a_1, a_2, \lambda; F) = 0, g_3(a_1, a_2, \lambda; F) = 0]$ be the top three equations of the system in Proposition 1 (after imposing $\hat{a} = a$ in equilibrium). We can derive the marginal effect of information acquisition
cost $F$ on the equilibrium choice variables $a_1, a_2, \lambda$ as

$$
\begin{bmatrix}
\frac{da_1}{dF} \\
\frac{da_2}{dF} \\
\frac{d\lambda}{dF}
\end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial g_1}{\partial a_1} & \frac{\partial g_1}{\partial a_2} & \frac{\partial g_1}{\partial \lambda} \\
\frac{\partial g_2}{\partial a_1} & \frac{\partial g_2}{\partial a_2} & \frac{\partial g_2}{\partial \lambda} \\
\frac{\partial g_3}{\partial a_1} & \frac{\partial g_3}{\partial a_2} & \frac{\partial g_3}{\partial \lambda}
\end{bmatrix}^{-1}
\begin{bmatrix}
\frac{\partial g_1}{\partial F} \\
\frac{\partial g_2}{\partial F} \\
\frac{\partial g_3}{\partial F}
\end{bmatrix},
$$

from which we obtain the following result.

**Proposition 2** As investors’ information acquisition cost $F$ is lower,

(i) more investors choose to be informed (i.e., higher $\lambda$) and the price is more informative;

(ii) the manager lowers his mean-improving effort $a_1^*$ and increases risk-reducing effort $a_2^*$;

(iii) market depth, measured by $C^{-1}$, increases.

Proposition 2-Part (i) is fairly intuitive and consistent with the analysis from single-task benchmark: more investors choose to acquire information when the cost of doing so is lower. Part (ii) is surprising to us. Even thought the manager’s two actions $a_1$ and $a_2$ are assumed to be equally costly, the proposition shows that lowering $F$ always incentivizes the manager to direct his actions towards reducing the underlying risk of the project, which occurs at the cost of lowering the mean return of the company. Because both actions are unobservable and are determined simultaneously with the investors’ information choice $\lambda$, we use a heuristic argument to help understand Proposition 2 - Parts (i) and (ii).

Recall from Lemma 1 the linear pricing coefficient $B = \frac{\tau_p + \lambda(\tau_c + \tau_\theta)}{\lambda\tau_c + \tau_\theta + \tau_p}$. We differentiate $B$ with respect to $F$ and apply chain rule to obtain:

$$
\frac{dB}{dF} = \frac{\partial B}{\partial \lambda} \frac{d\lambda}{dF} + \frac{\partial B}{\partial \tau_\theta} \frac{d\tau_\theta}{dF} \frac{da_2}{dF}.
$$

First, it is easy to verify $\frac{\partial B}{\partial \lambda} > 0$ and $\frac{\partial B}{\partial \tau_\theta} < 0$ given $B = \frac{\tau_p + \lambda(\tau_c + \tau_\theta)}{\lambda\tau_c + \tau_\theta + \tau_p}$. The thinking behind $\frac{\partial B}{\partial \lambda} > 0$ is intuitive. Having more informed investors $\lambda$ implies more people base their trade
on θ rather than p, which is only a noisy signal about θ. The result is that the (aggregated) trading reflects more about θ, making the market price more sensitive to a change in the underlying θ. Therefore, a higher λ tends to increase the pricing coefficient B. In contrast, when investors’ prior beliefs are stronger (i.e., a higher τθ), they will pay less attention to the arrival of new information in submitting their trades. The result is that the aggregated trade will depend more heavily on investors’ prior beliefs, which explains the $\frac{\partial B}{\partial \tau_\theta} < 0$. Second, we know from the two first-order conditions (FOCa1) and (FOCa2) that the coefficient $B = a_1 + a_2 = \frac{1}{\rho_M}$ is a constant, i.e., $\frac{dB}{dF} = 0$ in the model. Therefore, we can use (15) to derive the following:  

$$\frac{d\lambda}{dF} \propto \frac{da_2}{dF}.$$ (16)

That is, as the information acquisition cost F becomes lower, the percentage of informed investors λ and the manager’s risk-reducing effort $a_2$ either both increase or both decrease. However, we can rule out (by contradiction) the possibility that both λ and $a_2$ decrease for a lower F. To see the argument, note that the value of information to the investor is

$$\sqrt{\lambda^2(\Sigma - \hat{a}_2) + \rho^2 \sigma_\epsilon^2 \sigma_\theta^2 (\sigma_\epsilon^2 + \Sigma - \hat{a}_2)} \frac{\lambda^2 (\Sigma - \hat{a}_2) + \rho^2 \sigma_\epsilon^4 \sigma_\theta^2}{\lambda^2 (\Sigma - \hat{a}_2) + \rho^2 \sigma_\epsilon^2 \sigma_\theta^2},$$

which will be strictly higher if either λ or $\hat{a}_2$ is lower. Therefore, it violates the investor’s indifferent condition to have both λ or $\hat{a}_2$ decreasing (hence, a higher value of information) when the cost of information acquisition F is lower.

It is worthwhile discussing Proposition 2 - Part (iii) because the monotonic increase in the market depth is qualitatively different what one would derive from the single-task benchmark. In fact, Grossman and Stiglitz (1980) stated in their conjecture: “markets will be thinker under those conditions in which the percentage of individuals who are informed (λ) is either near zero or near unity.” While Grossman and Stiglitz suggest a non-monotonic relationship

---

4We use the fact $\frac{\partial B}{\partial \lambda} > 0$ and $\frac{\partial B}{\partial \tau_\theta} < 0$ shown in the paragraph and the observation that $\frac{\partial \hat{a}_2}{\partial a_2} > 0$ because $a_2$ reduces prior variance $\sigma_\theta^2$. 

---

17
between \( \lambda \) and market depth, a higher \( \lambda \) monotonically increases the market depth in our model. The difference is again driven by the fact that the manager can engage in risk-reducing activities. Therefore, the “prior” variance \( \sigma_\theta^2 \) that prior literature takes as given, is endogenously determined in a manager’s multi-dimensional activities. To highlight the role of the manager’s endogenous risk-reducing effort \( a_2 \), we investigate the derivative as follows:

\[
\frac{dC}{dF} = \frac{\partial C}{\partial \lambda} \frac{d\lambda}{dF} + \frac{\partial C}{\partial \tau_\theta} \frac{d\tau_\theta}{dF} \frac{da_2}{dF},
\]

where \( \frac{d\lambda}{dF} \) and \( \frac{da_2}{dF} \) are derived from the implicit function theorem (14). It is easy to verify that \( \frac{\partial C}{\partial \lambda} \) is non-monotonic. That is, if the manager were assumed to be unable to affect the underlying riskiness of the firm (i.e., \( \tau_\theta \) is independent of manager’s action), we would conclude the same non-monotonic relation between \( \lambda \) and market depth. It is the second term \( \frac{\partial C}{\partial \tau_\theta} \frac{d\tau_\theta}{dF} \frac{da_2}{dF} > 0 \) in (17) that ensures the overall unambiguous increasing relation.

### 5 Investor Welfare

We now turn the focus to studying investor welfare. In particular, we investigate whether a reduction in investors’ private information acquisition cost increase or decrease their ex-ante expected trading profit, net of the cost spent on acquiring private information if applicable. The answer is unclear a priori. While a lower \( F \) has a direct benefit of saving the cost of acquiring information, it also indirectly changes the very distribution of firm value as well as the percentage of the informed investors in the capital market (e.g., Proposition 2).

Note that one of the equilibrium conditions is that investors are indifferent from paying the cost to become informed and staying uninformed. Therefore, it is without loss of generality to analyze investors’ welfare only from the uninformed investors’ perspective because the informed investors earn the same expected payoff once we account for their acquisition cost. It is helpful to first analyze uninformed investors’ interim expected profit \( E(U|p) \) from
trading, that is, the expected profit after observing the price \( p \). It is well-known that the investor’s demand given the market price \( p \) is
\[
d = E(v|p) - p - \rho \var(v|p).
\]
Therefore, the uninformed investors’ wealth at the end of game is
\[
E(v|p) - p - \rho \var(v|p) \times (v - p),
\]
from which we can derive the following interim payoff:
\[
E(U|p) = -\exp\left\{-\frac{(E(v|p) - p)^2}{2 \var(v|p)}\right\}.
\]
(18)

As is clear from (18) above, investors earn profits by speculating the difference between price \( p \) and their expected firm value \( E(v|p) \), adjusted by the risk they bear \( \var(v|p) \). Ex ante, price is a random variable as it is a function of \( \theta \) and \( \epsilon \) (recall Lemma 1), and we integrate (18) over \( p \) in order to obtain investors’ ex ante payoff. Note that \( E(v|p) - p \) is normally distributed and, hence, the interim payoff (18) involves a chi-squared distribution. Using the moment generating function a non-central chi-squared distribution, we show that both the informed and uninformed investors earn an ex ante payoff \( E[U] = E[E[U|p]] \) as follows: (Derivation is in the proof of Proposition 3.)
\[
E[U] = -\left(\sqrt{1 + \frac{\var[E(v|p) - p]}{\var(v|p)}}\right)^{-1}.
\]
(19)

Investors’ ex ante payoff \( E[U] \) takes a surprisingly simple form and it has a one-to-one increasing relation to the volatility of \( E(v|p) - p \) adjusted for the risks they bear in equilibrium, i.e., the per unit-of-risk volatility \( R \approx \frac{\var[E(v|p) - p]}{\var(v|p)} \). The ratio makes it clear that investors benefit from reducing the ex-post uncertainties \( \var(v|p) \) they face; nevertheless, they benefit from having a higher ex-ante volatilities \( \var[E(v|p) - p] \). The different preferences towards ex-ante vs. ex-post uncertainties underlines the fundamental trade-offs in analyzing investors welfare. One can use the pricing function derived in Lemma 1 to further simplify the ratio \( R \) as:
\[
R = \frac{\var[E(v|p) - p]}{\var(v|p)} = \frac{\rho^2 \tau_\theta (\tau_\theta + \tau_p + \tau_p)}{\tau_e \tau_\epsilon (\tau_\theta + \tau_p + \lambda \tau_e)^2},
\]
(20)
where $\tau_\theta = 1/(\Sigma - a_2)$ is prior precision of the firm value $\theta$, and $\tau_p = (\lambda \tau_c / \rho)^2 \tau_\epsilon$ is the precision of a normally distributed signal of $\theta$ that is informationally equivalent to observing the price $p$.\(^5\)

Inspecting (19) and (20), we know that a lower information acquisition cost $F$ affects investors payoff via two effects. First, a lower $F$ changes the capital market composition by motivating more investors to become informed. Two, a lower $F$ changes the underlying risk of the firm by pressing the manager to pursue projects with more predictable cash flows. The following derivation summarizes the two effects on investor’s payoff

$$
\frac{dE[U]}{dF} \propto \frac{dR}{dF} = \frac{\partial R}{\partial \lambda} \frac{d\lambda}{dF} + \frac{\partial R}{\partial \tau_\theta} \frac{d\tau_\theta}{dF} \frac{da_2}{dF}.
$$

\(^{(21)}\)

It is easy to verify that the capital market effect $\frac{\partial R}{\partial \lambda} \frac{d\lambda}{dF}$ in (21) is positive.\(^6\) That is, a decrease in information acquisition cost $F$ has a direct effect of lowering investors’ welfare by motivating more investors to become informed in the capital market. The result may appear surprising at first because a higher $\lambda$ improves price informativeness (i.e., higher $\tau_p$) and, hence, reduces the posterior uncertainties $\text{var}(v|p)$ faced by uninformed investors. However, a higher fraction $\lambda$ of informed investors also synchronizes the uninformed investors’ posterior belief $E[v|p]$ more closely with the price $p$, reducing the ex ante opportunity to trade and benefit from the deviation $E(v|p) - p$ (i.e., lower $\text{var}[E(v|p) - p]$). In fact, higher $\lambda$ reduces the $\text{var}[E(v|p) - p]$ so much that its effects in lowering investors’ welfare dominates the effect of $\lambda$ in reducing the risk $\text{var}(v|p)$ that individual investors’ bear ex post.

We know from (21) that the potential for investors to benefit from a lower $F$ can only arise internally from the firm side, specifically, the type of projects the firm takes. Recall that reducing $F$ motivates a higher risk-reducing effort $a_2$ (and, hence, increases $\tau_\theta = 1/(\Sigma - a_2)$). Investors can potentially benefit from the reduced fundamental riskiness if $\frac{\partial R}{\partial \tau_\theta} > 0$.

\(^5\) $\tau_p$ is the precision of the normally distributed variable $\frac{p - A}{B} = \theta - \frac{\rho}{\lambda \tau_c} \epsilon$.

\(^6\) This can be seen by recalling $\frac{d\lambda}{dF} < 0$ from Proposition 2 and by verifying $\frac{\partial R}{\partial \lambda} < 0$ in (21).
Proposition 3 - Part (ii) below shows that investors benefit from a lower $F$ only when the percentage of informed investors $\lambda$ is already large enough.

**Proposition 3 (Welfare)** Investor welfare is measured by ex-ante expected trading profit (net of information acquisition cost $F$, if applicable) prior to observing market price or any signal. In equilibrium,

(i) both informed and uninformed investors’ welfare is

$$E[U] = -\left(\sqrt{1 + \frac{\text{var}[E(v|p) - p]}{\text{var}(v|p)}}\right)^{-1}. \quad (22)$$

(ii) A lower information acquisition cost $F$ reduces investor welfare $E[U]$ if and only if the percentage $\lambda$ of informed investors is less than a uniquely characterized threshold $\lambda^*$. 

**Corollary 2** For $\lambda^* \in (0, 1)$, lowering $F$ first decreases and then increases investors welfare.

To illustrate the conditions under which investors benefit from the manager’s investment in safer projects (i.e., higher $\frac{\partial R}{\partial \theta} > 0$), we rewrite the pricing function in Lemma 1 as follows:

$$p = \frac{\lambda\tau_I}{\lambda\tau_I + (1 - \lambda)\tau_U} E[v|\theta] + \frac{(1 - \lambda)\tau_U}{\lambda\tau_I + (1 - \lambda)\tau_U} E(v|p) - \frac{\rho}{\lambda\tau_I + (1 - \lambda)\tau_U} \epsilon, \quad (23)$$

where $\tau_I = \text{var}^{-1}(v|\theta)$ and $\tau_U = \text{var}^{-1}(v|p)$ are the posterior precision of informed and uninformed investors, respectively. That is, the market-clearing price equals to the aggregate beliefs $\bar{E}$ among investors (weighted by precision and mass) minus the risk premium required by investors to clear the random supply. Rewriting the pricing function as in (2) helps analyze the sources of volatility $\text{var}[E(v|p) - p]$ and, hence, the ratio $R$ in (20). In particular,

$$R = \frac{\text{var}[E(v|p) - p]}{\text{var}(v|p)} = \tau_U \times \text{var} \left[ \bar{E} - E(v|p) - \frac{\rho \epsilon}{\lambda\tau_I + (1 - \lambda)\tau_U} \right]. \quad (24)$$
The riskiness of the project $\tau_\theta$ affects $R$ (hence, investors’ payoffs) via two channels: the volatility of difference in beliefs $\text{var} [\bar{E} - E(v|p)]$, and that of the risk adjustment $\text{var} \left[ \frac{\rho \epsilon}{\lambda \tau_t + (1-\lambda) \tau_U} \right]$ (net of the covariance between them).

We start by showing that the manager’s chase of safer project (i.e., higher $\tau_\theta$) lowers $\tau_U \times \text{var} [\bar{E} - E(v|p)]$ part of (24). This can be seen by noting

$$E(v|p) = \frac{\tau_\theta}{\tau_\theta + \tau_p} \hat{\alpha}_1 + \frac{\tau_p}{\tau_\theta + \tau_p} \frac{p - A}{B} = \frac{\tau_\theta}{\tau_\theta + \tau_p} \hat{\alpha}_1 + \frac{\tau_p}{\tau_\theta + \tau_p} (\theta - \frac{\rho}{\lambda \tau_\epsilon}).$$

(25)
and hence,

\[
\tau_U \times \text{var} [\bar{E} - E(v|p)] = \left( \frac{\lambda \tau_I}{\lambda \tau_I + (1 - \lambda) \tau_U} \right)^2 \frac{\tau_U}{\tau_p + \tau_\theta} = \frac{\tau_u \lambda^2 (\tau_\theta + \tau_p + \tau_e)}{(\tau_\theta + \tau_p + \lambda \tau_e)^2}. \tag{26}
\]

The finding that \(\text{var} [\bar{E} - E(v|p)] / \text{var} (v|p)\) decreases in \(\tau_\theta\) is not surprising. As shown in (2), the difference between the aggregated belief \(\bar{E}\) and the uninformed investors’ posterior \(E(v|p)\) is caused by the presence of the informed investors, whose precision-adjusted mass is \(\frac{\lambda \tau_I}{\lambda \tau_I + (1 - \lambda) \tau_U}\). Higher \(\tau_\theta\) improves the uninformed investors’ precision \(\tau_U\) while keeping the informed investors’ precision \(\tau_I\) unchanged. The result is that uninformed investors will trade more, which reduces the precision-adjusted mass of the informed investors, i.e., lowering \(\frac{\lambda \tau_I}{\lambda \tau_I + (1 - \lambda) \tau_U}\). Since the presence of informed investors is what causes the aggregated belief \(\bar{E}\) to deviate from uninformed investors’ belief \(E(v|p)\) in the first place, the difference between \(\bar{E}\) and \(E(v|p)\) diminishes (and hence a diminishing \(\text{var} [\bar{E} - E(v|p)]\)) as a higher \(\tau_\theta\) reduces the relative mass of informed investors.

The benefit to investors for trading a safer asset (i.e., \(\tau_\theta\)) comes from \(\tau_U \times \text{var} [\frac{\rho \epsilon}{\lambda \tau_I + (1 - \lambda) \tau_U}]\) part of (24) (and its related covariance term \(-\tau_U \text{cov} (\frac{\rho \epsilon}{\lambda \tau_I + (1 - \lambda) \tau_U}, \bar{E} - E(v|p))\)). We illustrate the intuition for \(\tau_U \times \text{var} [\frac{\rho \epsilon}{\lambda \tau_I + (1 - \lambda) \tau_U}]\) and the similar argument applies to the covariance term. Note that

\[
\tau_U \times \text{var} [\frac{\rho \epsilon}{\lambda \tau_I + (1 - \lambda) \tau_U}] = \rho^2 \sigma^2 \times \frac{\tau_U}{[\lambda \tau_I + (1 - \lambda) \tau_U]^2}. \tag{27}
\]

One can verify that (27) increases in \(\tau_\theta\) if and only if the percentage of informed investors \(\lambda\) is sufficiently large. This can be seen by analyzing the two extreme cases: \(\lambda \to 1\) and \(\lambda \to 0\). For \(\lambda \to 1\), it follows from (27) that \(\tau_U \times \text{var} [\frac{\rho \epsilon}{\lambda \tau_I + (1 - \lambda) \tau_U}]\) converges to \(\rho^2 \sigma^2 \tau_U / \tau_I^2\), which increases in \(\tau_\theta\) because a more precise prior \(\tau_\theta\) improves uninformed investors’ posterior precision \(\tau_U\) without affect informed investors’ posterior \(\tau_I\). In contrast, \(\tau_U \times \text{var} [\frac{\rho \epsilon}{\lambda \tau_I + (1 - \lambda) \tau_U}]\)
converges to $\rho^2 \sigma_t^2 \frac{1}{\tau_U}$ if $\lambda \to 0$, which unambiguously decreases in $\tau_\theta$. Intuitively, the reason $\lambda$ matters in analyzing the effect of $\tau_\theta$ is that a more precise prior (i.e., higher $\tau_\theta$) improves uninformed investors’ posterior precision $\tau_U$ more than it does to the average precision $\lambda \tau_I + (1 - \lambda) \tau_U$. This occurs because the informed investors’ posterior precision $\tau_I$ is independent of $\tau_\theta$ and, therefore, a higher $\tau_\theta$ increases the average precision only at a discounted rate $(1 - \lambda)$. When the informed investors’ representation $\lambda$ is high, motivating the manager to reduce the underlying risk of the firm (i.e., higher $\tau_\theta$) increasingly favors the uninformed investors’s posterior precision $\tau_U$ relative to that of the average precision. For sufficiently large $\lambda$, the relative increase in precision of $\tau_U$ becomes the dominate effect in determining the net effect of $\frac{\tau_U}{\lambda \tau_I + (1 - \lambda) \tau_U}$ even though the denominator increases in a quadratic manner. This explains why we need large $\lambda$ in Proposition 3 under which lowering information cost $F$ benefits investors ex ante.

6 Value of Public Disclosure

In this section, we introduce firm’s disclosure and illustrate the value of public disclosure in the current framework. We derive the optimal disclosure quality to maximize the expected firm value. We also investigate how the optimal quality change in response to technological innovations that lowers investor’s private information acquisition cost.

The timeline of the game is the same as we studied above, with an addition that all investors will observe a publicly disclosed report $y = \theta + \phi$ prior to submitting their orders. The noise term $\phi$ contained in the public report is normally distributed with mean zero and precision $\tau_y = 1/\sigma_y^2$. The disclosure quality $\tau_y$ is chosen publicly at the beginning of the game. The next lemma summarizes the subgame equilibrium for a given disclosure precision $\tau_y$. We specify the linear pricing function in the lemma and leave the system of equations used to determine the equilibrium actions $(a_1, a_2, \lambda)$ to the appendix for the equations are similar to those specified in Proposition 1.
Lemma 2  Fixing any disclosure precision $\tau_y \geq 0$, the linear pricing function is

$$p = \alpha_0 + \alpha_\theta \times \theta + \alpha_y \times y - \alpha_\epsilon \times \epsilon,$$

(28)

where the non-negative coefficients $\alpha_0, \alpha_\theta, \alpha_y, \alpha_\epsilon$ are shown in the appendix. The manager’s actions $(a_1, a_2)$ and the percentage $\lambda$ of informed investors are uniquely determined as a function of $\tau_y$.

Having characterized the subgame above, we can endogenize the optimal disclosure quality $\tau_y^*$ that maximizes the expected firm value $E[v]$. Note that we assume away any direct cost of improving disclosure quality to highlight the fact that higher quality can have an endogenous cost of lowering the firm value beyond some point.

Proposition 4 (Endogenous Disclosure Quality) There is a unique interior disclosure precision choice $\tau_y^*$ that maximizes the expected firm value $E[v]$. At the optimal precision $\tau_y^*$, the manager chooses $a_1^* = \frac{\tau_y^* + \tau_p + \lambda(\tau_e + \tau_y)}{\tau_y^* + \lambda \tau_e + \tau_y + \tau_p}$ and $a_2^* = 0$, where $\tau_p = (\lambda \tau_e / \rho)^2 \tau_e$ and the percentage of informed investors $\lambda$ is uniquely determined by

$$\exp(\rho \ast F) = \sqrt{1 + \frac{\tau_e}{\tau_y^* + (\lambda \tau_e / \rho)^2 \tau_e + 1 / \Sigma}}.$$  

Moreover, the optimal disclosure $\tau_y^*$ is higher when it is less costly for investors to acquire private information. That is,  

$$\frac{d\tau_y^*}{dF} < 0.$$  

We illustrate the optimal disclosure quality $\tau_y^*$ in Figure 3. It is clear from the figure that the manager’s value-improving effort $a_1$ and, hence, the expected firm value $E[v]$ is unimodal in the public disclosure precision: it increases in $\tau_y$ for $\tau_y \leq \tau_y^* = 0.102$ and decreases for $\tau_y > \tau_y^*$. (We prove the unimodality in the appendix.) To understand the unimodality result, first note that, on both sides of $\tau_y^*$, improving disclosure quality $\tau_y$
reduces the fraction $\lambda$ of investors who choose to become informed in equilibrium. This is intuitive as better public disclosure directly lowers the uncertainties faced by any investors, which lowers the incentives to become an informed investor by reducing the informational advantage that informed investors have over uninformed ones. The fact that anticipated public disclosure reduces individual investors’ incentive to acquire private information is well-understood results in prior literature (e.g., Chen et al., 2017; Diamond, 1985; Demski and Feltham, 1994; McNichols and Trueman, 1994; Fischer and Stocken, 2010; Gao and Liang, 2013; Han and Yang, 2013; Amador and Weill, 2010).

Why does disclosure quality have qualitatively different effects on the two sides of $\tau_y^*$, even though a higher $\tau_y$ has the same effect of lowering private information acquisition $\lambda$?
The key to understanding the difference is that the manager’s risk-reduction effort \( a_2 \) stays at zero for \( \tau_y \geq \tau_y^* \). In other words, the nature of the manager’s incentive problem qualitatively changes from multi-dimensional (i.e., value-and-risk tradeoff) to single-dimensional as the disclosure quality increases above the threshold \( \tau_y^* \). For \( \tau_y < \tau_y^* \), improving disclosure quality is value-enhancing in the sense that it induces the manager to shift attention away from the risk-reducing effort \( a_2 \) into the value-improving effort \( a_1 \). Here, the fact using disclosure to crowd out private incentives to acquire information (i.e., lower \( \lambda \)) is valuable because, as we have shown in Proposition 2, a higher fraction of informed investor has a side effect of disproportionately pressures the manager to focus on risk-reducing effort. By credibly convincing less investors to become informed, more precise public disclosures take the pressure (of chasing lower risk-return project) off from the manager and, therefore, restores the manager’ incentive to take on projects with higher expected returns.

Improving disclosure quality beyond \( \tau_y^* \) is value-decreasing. The thinking is that, when the manager’s job is only one dimensional (i.e., to improve firm value in the sense of first-order stochastic dominance), the direct benefit of disclosure in motivating the manager’s unobservable effort is outweighed by the indirect effect that disclosure has in crowding out investors’ private information acquisition. To see this, we can write the first-order condition used to determine the manager’s optimal value-enhancing effort \( a_1 \) (recall \( a_2 = 0 \) for \( \tau_y > \tau_y^* \)):

\[
a_1 = MB = \frac{\tau_y + \tau_p + \lambda(\tau_e + \tau_\theta)}{\tau_y + \lambda\tau_e + \tau_\theta + \tau_p}.
\]

(29)

It follows from (29) that, all else equal, increasing either disclosure quality \( \tau_y \) or the percentage of informed investors \( \lambda \) will incentivize the manager to exert a higher effort, i.e., \( \frac{\partial MB}{\partial \tau_y} > 0 \) and \( \frac{\partial MB}{\partial \lambda} > 0 \). Differentiating the manager’s marginal benefit (29) with respect to \( \tau_y \), we characterize the effect of a higher disclosure quality on the manager’s equilibrium...
effort as
\[
\frac{da_1}{d\tau_y} = \frac{\partial MB}{\partial \tau_y} + \frac{\partial MB}{\partial \lambda} \frac{\partial \lambda}{\partial \tau_y} < 0. \tag{30}
\]

An improved disclosure has two countervailing effects on the manager’s value-improving effort. On the one hand, more precise disclosure helps directly reveal the firm’s true value to the investors and, hence, ties the equilibrium price more closely to manager’s true effort. On the other hand, a higher \( \tau_y \) lowers the fraction \( \lambda \) of informed investors. A lower \( \lambda \) reduces the information contained in the informed investors’ orders and hence the information \( \tau_p = (\lambda \tau_e / \rho)^2 \tau_e \) revealed from price. We show in the appendix that the indirect effect caused by a reduction of \( \lambda \) dominates and, therefore, the equilibrium sensitivity between the manager’s effort and the market price goes down as one improves disclosure quality \( \tau_y \). The analysis demonstrates an endogenous cost of disclosure in terms of lowering the expected firm value.

A takeaway from Proposition 4 is that improving public disclosure is an efficient way to solve the prisoner’s dilemma between investors’ private information acquisition and the manager’s chase of low value-risk projects. The following corollary shows another benefit of public disclosure. That is, not only does public disclosure improves the firm value for \( \tau_y \in [0, \tau_y^*] \), it also improves the trading profit investors expect to earn ex ante.

**Corollary 3** Improving disclosure quality for \( \tau_y \in [0, \tau_y^*] \) strictly increases investors ex-ante trading profit \( E[U] \) as shown in (19).

### 7 Conclusion

Technological innovations such as telephone, e-mails, and social media have lowered the cost for investors to discover and communicate private information. For example, recent studies document that the decreasing cost made it cost efficient for asset management companies to
collect real-time satellite images and consumer transaction data for valuation purposes. Empirical evidence suggests that the introduction of these technological innovations increases price informativeness and market liquidity through decreased information acquisition cost (e.g., Blankespoor et al., 2014; Zhu, 2019). The general belief is that, by encouraging private information discoveries, technologies that lower information acquisition cost serves a governance mechanism that disciplines the manager’s action.

The idea that technological innovations can be powerful governance mechanism is shared among practitioners. In an open letter signed by its CEO and founder Mark Zuckerberg shortly before Facebook went public in 2012, Zuckerberg stated that, we often talk about inventions like the printing press and the television by simply making communication more efficient, they led to a complete transformation of many important parts of society. ... They encouraged progress.”

In this paper, we explicit model individual investors’ information acquisition and examine the disciplinary role of market price for managerial decisions as one lowers the cost of information acquisition. While our results echo empirical findings that a lower information acquisition cost improves price informativeness as well as market liquidity, we show that existing understanding left out some important cost considerations. In particular, we show that more active private information discoveries by investors disproportionally encourages the manager to engage in activities aimed to lower the riskiness of the project at the cost of lowering firm value on expectation. Private information discoveries distort the manager’s risk-value tradeoff in the way that they not only lower the expected firm value, but can also make investors strictly worse off even taking into account their risk aversion. The result suggests that a decrease in information acquisition is like a a prisoner’s dilemma. It is true that lowering information acquisition cost makes each individual investor better off ex post. Collectively, however, the lower cost incentivizes too many investors to acquire information, which not only pressures the manager to take on lower return projects but also irons out ex-ante price variations and, hence, destroys the expected trading profit.
We also investigate the role of public disclosure in mitigating the “prisoner’s dilemma”. The result shows that providing public disclosure is an effective way to restore the manager’s distorted incentives to chase low risk-and-value projects. There exists a unique interior disclosure quality that maximizes the expected firm value. We show that improving disclosure quality towards the optimal level not only increases the expected firm value but also the trading profits investors expected to gain from the capital market.
References


A Appendix

Proof of Lemma 1. The equilibrium is solved using backward induction. We first fix the market conjecture $\hat{a}_1$ about manager’s effort choice as given and solve for the trading subgame. Solving the trading subgame given $\hat{a}_1$ is standard. We follow the similar steps as in Grossman and Stiglitz (1980) to derive the following pricing function

$$p = A + B \times ñ - C \times \epsilon,$$  \hfill (A.1)

where the non-negative coefficients are

$$A = \frac{(1-\lambda)\tau_e}{\lambda \tau_e + \tau_\theta + \tau_p} a^*, \quad B = \frac{\tau_p + \lambda(\tau_e + \tau_\theta)}{\lambda \tau_e + \tau_\theta + \tau_p}, \quad C = \frac{\rho(\tau_e + \tau_\theta + \tau_p/\lambda)}{\tau_e(\lambda \tau_e + \tau_\theta + \tau_p)},$$

and $\tau_p = (\lambda \tau_e/\rho)^2 \tau_\epsilon$ is the precision of price used as a signal of $\theta$. In equilibrium, the fraction $\lambda$ of informed investors is $\lambda = 1$ for $F < F_0$ and $\lambda = 0$ for $F > \bar{F}$, where $F_0 = \frac{1}{\rho} \log \left( \sqrt{\frac{\Sigma^2 \sigma^2 + \Sigma + \rho^2 \sigma^4 \sigma^2}{\lambda^2 \Sigma + \rho^2 \sigma^4 \sigma^2}} \right)$ and $\bar{F} = \frac{1}{\rho} \log \left( \sqrt{\frac{\rho^2 \sigma^2 \sigma^2 (\sigma^2 + \Sigma)}{\rho^2 \sigma^4 \sigma^2}} \right)$. For $F \in [F_0, \bar{F}]$, $\lambda$ is uniquely determined by the following equation

$$\exp(\rho \circ F) = \sqrt{\frac{\lambda^2 \Sigma + \rho^2 \sigma^2 \sigma^2 (\sigma^2 + \Sigma)}{\lambda^2 \Sigma + \rho^2 \sigma^4 \sigma^2}}.$$  \hfill (A.2)

Having solved the pricing subgame above, we then derive the manager’s best response as a function of the market conjecture and, hence, the pricing function. It is easy to verify that the first-order-condition that determines the manager’s best response is

$$a_{BR}(\hat{a}_1) = B = \frac{\tau_p + \lambda(\tau_e + \tau_\theta)}{\lambda \tau_e + \tau_\theta + \tau_p}. \hfill (A.3)$$

Since the pricing coefficient $B$ is independent of the investor’s conjecture $\hat{a}$, we know that the manager choosing $a_{BR} = B$ shown above is a dominant strategy in the sense that it is independent of the market conjecture $\hat{a}$. Substituting $a^* = B$ into the linear pricing function completes the proof.

Proof of Corollary 1. Part (i) of the corollary follows directly from (A.2). Given that a
lower $F$ results in a higher $\lambda$, we prove Part (ii) by noting (A.3) that $a^*$ increases in $\lambda$.

**Proof of Proposition 1.** Investors do not observe the manager’s actions and, hence, form their beliefs $\hat{a}_1, \hat{a}_2$ the two effort. Given the conjectures, the distribution of the firm value is fixed and the subsequent trading subgame is characterized similar to that in Lemma 1. The complication is that all the linear pricing coefficients seen in (A.1) are functions of the conjectured risk-reducing effort $\hat{a}_2$ because $\sigma_\theta^2 = \Sigma - a_2$.

Following the similar steps in the proof of Lemma 1, we characterize the manager’s best responses $a_1^{BR}$ and $a_2^{BR}$ as a function of the pricing function as well as investors’ conjectured $\hat{a}_1$ and $\hat{a}_2$. In particular, the two first-order conditions for $a_1^{BR}$ and $a_2^{BR}$ are, respectively

$$B(\hat{a}_1, \hat{a}_2) = a_1^{BR} + a_2^{BR},$$  \hspace{1cm} (FOCa$_1$)

and, similarly, the first-order condition for $a_2^{BR}$ is

$$\rho M B^2(\hat{a}_1, \hat{a}_2) = a_1^{BR} + a_2^{BR}.$$  \hspace{1cm} (FOCa$_2$)

The investors’ indifferent condition to determine the percentage of informed traders is

$$\exp(\rho \ast F) = \sqrt{\frac{\lambda^2(\Sigma - \hat{a}_2) + \rho^2 \sigma_\epsilon^2 \sigma_\epsilon^2 (\sigma_\epsilon^2 + \Sigma - \hat{a}_2)}{\lambda^2(\Sigma - \hat{a}_2) + \rho^2 \sigma_\epsilon^2 \sigma_\epsilon^2}},$$  \hspace{1cm} (Indifference)

The equilibrium is then characterized by (FOCa$_1$), (FOCa$_2$), (Indifference), and two rational expectation conditions $a_1 = \hat{a}_1$ and $a_2 = \hat{a}_2$. Collecting conditions verifies the proposition.

**Proof of Proposition 2.** Denote by $[g_1(a_1, a_2, \lambda; F) = 0, g_2(a_1, a_2, \lambda; F) = 0, g_3(a_1, a_2, \lambda; F) = 0]$ be the system of three equations (FOCa$_1$), (FOCa$_2$), and (Indifference) in Proposition 1 that characterizes the equilibrium (after imposing rational expectation in equilibrium, i.e., $\hat{a} = a$). That is, $g_1 = 0$ and $g_2 = 0$ are the first-order conditions for $a_1$ and $a_2$, respectively,
and \( g_3 = 0 \) is the investors indifferent condition used to determine the fraction \( \lambda \) of informed investors. Applying the implicit function theorem to the systems of equations, we determine the marginal effect of information acquisition cost \( F \) on all the equilibrium choice variables \( a_1, a_2, \lambda \):

\[
\begin{bmatrix}
\frac{da_1}{dF} \\
\frac{da_2}{dF} \\
\frac{d\lambda}{dF}
\end{bmatrix} = -
\begin{bmatrix}
\frac{\partial g_1}{\partial a_1} & \frac{\partial g_1}{\partial a_2} & \frac{\partial g_1}{\partial \lambda} \\
\frac{\partial g_2}{\partial a_1} & \frac{\partial g_2}{\partial a_2} & \frac{\partial g_2}{\partial \lambda} \\
\frac{\partial g_3}{\partial a_1} & \frac{\partial g_3}{\partial a_2} & \frac{\partial g_3}{\partial \lambda}
\end{bmatrix}
^{-1}
\begin{bmatrix}
\frac{\partial g_1}{\partial F} \\
\frac{\partial g_2}{\partial F} \\
\frac{\partial g_3}{\partial F}
\end{bmatrix}.
\]  

(A.4)

We omit the closed-form expression of the derivatives for brevity. It is sufficient, for the purpose of proving Parts (i) and (ii) of the proposition, to show the following:

\[
\frac{da_1}{dF} \times \frac{d\lambda}{dF} < 0,
\]  

(A.5)

\[
\frac{da_2}{dF} \times \frac{d\lambda}{dF} > 0.
\]  

(A.6)

That is, the sign of \( \frac{da_1}{dF} \) is opposite to the sign of \( \frac{d\lambda}{dF} \), while \( \frac{da_2}{dF} \) shares the same sign of \( \frac{d\lambda}{dF} \).

This means that as the cost of information acquisition \( F \) goes down, either (i) both \( a_2 \) and \( \lambda \) increase in equilibrium, or (ii) they both decrease in equilibrium. We rule out the second possibility by contradiction. Suppose, by contradiction that both \( a_2 \) and \( \lambda \) decreases when \( F \) becomes smaller. We know from the proof of Lemma 1 that the value of information to an investor is

\[
value = \sqrt{\frac{\lambda^2 (\Sigma - \hat{a}_2) + \rho^2 \sigma^2 \sigma^2_e (\sigma^2_e + \Sigma - \hat{a}_2)}{\lambda^2 (\Sigma - \hat{a}_2) + \rho^2 \sigma^4 \sigma^2_e}}.
\]

It is easy to verify that the value of information increases in prior variance \( \sigma^2_\theta = \Sigma - a_2 \) and decreases in \( \lambda \). Therefore, an decrease in both \( a_2 \) and \( \lambda \) will strictly increases the value of information to an investor. However, this contradicts the premise that the cost of information \( F \) is lower because the equilibrium indifference condition \( \exp(\rho * F) = \sqrt{\frac{\lambda^2 (\Sigma - \hat{a}_2) + \rho^2 \sigma^2 \sigma^2_e (\sigma^2_e + \Sigma - \hat{a}_2)}{\lambda^2 (\Sigma - a_2) + \rho^2 \sigma^4 \sigma^2_e}} \).
is violated. Ruling out the other alternative allows us to prove the following claim

\[
\frac{da_1}{dF} > 0, \quad \frac{da_2}{dF} < 0, \quad \frac{d\lambda}{dF} < 0.
\]

It remains to prove Part (iii) of the proposition that lowering \( F \) improves the market depth, which is measured by \( 1/C \). Recall that the pricing coefficient \( C = \frac{\rho(\tau_e + \tau_\theta + \tau_p/\lambda)}{\tau_e(\lambda\tau_e + \tau_\theta + \tau_p)} \) where \( \tau_\theta = \frac{1}{\Sigma-a_2} \). We apply the chain rule to show the following

\[
\frac{dC}{dF} = \frac{\partial C}{\partial \lambda} \frac{d\lambda}{dF} + \frac{\partial C}{\partial \tau_\theta} \frac{\partial \tau_\theta}{\partial a_2} \frac{da_2}{dF},
\]

where \( \frac{d\lambda}{dF} \) and \( \frac{da_2}{dF} \) are derived in (A.4) using the implicit function theorem. It is tedious but straightforward to show that \( \frac{dC}{dF} > 0 \). The claim that lower \( F \) improve market depth follows easily as the market depth is measured by \( 1/C \). ■

**Proof of Proposition 3.** Given an observed price \( p \), an uninformed investor’s *interim* expected payoff is

\[
E(U_i|p) = -\exp\{-\rho[E(W_i|p) - \frac{\rho}{2}\text{var}(W_i|p)]\},
\]

where \( W_i \) is Agent \( i \)'s final wealth. Assuming away without loss of generality investors’s initial endowment, we know that

\[
W_i = \frac{E(v|p) - p}{\rho \text{var}(v|p)} \times (v - p).
\]

Substituting (A.9) back to (A.8), one can rewrite \( E(U_i|p) \) as follows

\[
E(U_i|p) = -\exp\{-\frac{[E(v|p) - p]^2}{2 \text{var}(v|p)}\}.
\]

Next, we calculate Agent \( i \)'s *ex ante* payoff \( E[E(U_i|p)] \). Note first that \( E(v|p) - p \) is normally distributed from ex ante perspective as all variables are normally distributed and
the pricing function is linear. Therefore, if we define

\[ P = \text{var}[E(v|p) - p], \quad (A.11) \]
\[ Q = \frac{E(v|p) - p}{\sqrt{P}}, \quad (A.12) \]

then we know

\[ Q \sim N(0, 1). \quad (A.13) \]

To derive the zero mean in (A.13), we use the fact that the equilibrium price function (e.g., Lemma 1) is unbiased and, therefore, we can show \( E[E(v|p) - p] = 0 \).

One can rewrite (A.10) as

\[ E(U_i|p) = -\exp\{-\frac{P}{2 \text{var}(v|p)} \times Q^2\}. \quad (A.14) \]

To compute \( E[E(U_i|p)] \), which requires taking expectations of the exponential of the quadratic of a standard normal random variable \( Q \). We can use the moment generating function of a Chi-square distribution to show

\[ E[U] = E[E(U_i|p)] = -\frac{1}{\sqrt{1 + \frac{P}{\text{var}(v|p)}}}, \quad (A.15) \]

which is equivalent to

\[ E[U] = -\left(\sqrt{1 + \frac{\text{var}[E(v|p) - p]}{\text{var}(v|p)}}\right)^{-1}. \quad (A.16) \]

Let \( R := \frac{\text{var}[E(v|p) - p]}{\text{var}(v|p)} \). It follows from (A.16) that

\[ \frac{dE[U]}{dF} \propto \frac{dR}{dF} \]
Using the pricing function derived in Proposition 1 and Lemma 1, we can show that

\[ R = \frac{\text{var}[E(v|p) - p]}{\text{var}(v|p)} = \frac{\rho^2 \tau_\theta (\tau_\theta + \tau_p + \tau_\epsilon)}{\tau_\epsilon \tau_\epsilon (\tau_\theta + \tau_p + \lambda \tau_\epsilon)^2}. \]  

(A.17)

Applying chain rule, we obtain

\[ \frac{dR}{dF} = \frac{\partial R}{\partial \lambda} \frac{d\lambda}{dF} + \frac{\partial R}{\partial \tau_\theta} \frac{d\tau_\theta}{dF} \frac{da_2}{dF}, \]  

(A.18)

where \( \frac{d\lambda}{dF} \) and \( \frac{da_2}{dF} \) are derived in (A.4) using the implicit function theorem. Tedium algebra verifies the claim that \( \frac{dE[U]}{dF} > 0 \) if only only if \( \lambda < \lambda^* \).

**Proof of Corollary 2.** The corollary follows from Proposition 3 - Part (ii) and the fact that \( \frac{d\lambda}{dF} < 0 \) shown in Proposition 2.

**Proof of Lemma 2.** We guess and verify that the market-clearing price takes the form

\[ p = \alpha_0 + \alpha_\theta \times \theta + \alpha_y \times y - \alpha_\epsilon \times \epsilon. \]

Note that observing the price \( p \) is informationally equivalent to observing \( q = \frac{p - \alpha_0 - \alpha_y y}{\alpha_\theta} = \theta - \frac{\alpha_\epsilon}{\alpha_\theta} \epsilon \), and \( q \) is normally distributed with mean \( \theta \) and precision \( \tau_p = \left( \frac{\alpha_\theta}{\alpha_\epsilon} \right)^2 \tau_\epsilon \).

Denote by \( \tau_I = \text{var}^{-1}(v|\theta) \) and \( \tau_U = \text{var}^{-1}(v|p) \) the posterior precision of informed investors and uninformed investors, respectively. One can show that

\[ \tau_I = \tau_\epsilon \text{ and } \tau_U = \frac{1}{\frac{1}{\tau_\epsilon} + \frac{1}{\tau_\theta + \tau_y + \tau_p}}, \]  

(A.19)

where \( \tau_\theta = (\Sigma - \hat{a}_2)^{-1} \). An informed investor’s demand is \( d_I = \tau_I \frac{E(v|\theta)-p}{\rho} \) and an uninformed investor’s demand \( d_U = \tau_U \frac{E(v|p)-p}{\rho} \). The market-clearing condition is

\[ \lambda d_I + (1 - \lambda) d_U = \epsilon, \]  

(A.20)
from which we can derive the linear pricing coefficients as follows:

\[
\begin{align*}
\alpha_0 &= \frac{(1 - \lambda)\tau_\theta}{(\lambda \tau_e + \tau_\theta + \tau_y) + \tau_e (\frac{\alpha_0}{\alpha_e})^2 \hat{a}_1}, \\
\alpha_\theta &= \frac{(\lambda (\tau_e + \tau_\theta) + \tau_y) - (1 - \lambda)\tau_y + \tau_e (\frac{\alpha_0}{\alpha_e})^2}{(\lambda \tau_e + \tau_\theta + \tau_y) + \tau_e (\frac{\alpha_0}{\alpha_e})^2}, \\
\alpha_y &= \frac{(1 - \lambda)\tau_y}{(\lambda \tau_e + \tau_\theta + \tau_y) + \tau_e (\frac{\alpha_0}{\alpha_e})^2}, \\
\alpha_\epsilon &= \frac{(1 - \lambda)\rho \tau_e \epsilon + (\tau_e + \tau_\theta + \tau_y) + \tau_e (\frac{\alpha_0}{\alpha_e})^2}{\rho \tau_e (\lambda \tau_e + \tau_\theta + \tau_y) + \tau_e (\frac{\alpha_0}{\alpha_e})^2},
\end{align*}
\] (A.21) (A.22)

where \( r = 1/\rho \) is the investors’ risk tolerance.

Note that the coefficients above are not fully characterized unless we find the to-be-determined ratio \( \frac{\alpha_0}{\alpha_e} \). To do so, we divide (A.21) over (A.22) and equate it to \( \frac{\alpha_0}{\alpha_e} \), and prove that the equation has a unique real root

\[
\frac{\alpha_\theta}{\alpha_\epsilon} = \lambda r \tau_e. 
\] (A.23)

Substituting (A.23) back to the coefficients derived above characterizes the pricing function, given investors’ conjecture \( \hat{a}_1 \) and \( \hat{a}_2 \).

Taking the pricing function given, we can derive the manager’s best-response actions \( a_1^{BR} \) and \( a_2^{BR} \). The first-order condition for \( a_1^{BR} \) is

\[
\alpha_\theta + \alpha_y = a_1^{BR} + a_2^{BR}, 
\] (A.24)

and the first-order condition for \( a_2^{BR} \) is

\[
\rho_M (\alpha_\theta + \alpha_y) = a_1^{BR} + a_2^{BR}. 
\] (A.25)

The investors’ indifference condition that determines the equilibrium fraction of informed
investors $\lambda$ is

$$
\exp(\rho * F) = \sqrt{1 + \frac{\tau_e}{\tau_y + \tau_p + \tau_0}},
$$

(A.26)

where $\tau_0 = 1/(\Sigma - \hat{a}_2)$ and $\tau_p = (\lambda r \tau_e)^2 \tau_e$. Imposing rational expectations, we characterize the equilibrium using the system of equations: (A.24), (A.25), (A.26), $\hat{a}_1 = a_1$, and $\hat{a}_2 = a_2$ as in Proposition 1. ■

**Proof of Proposition 4.** Rewrite [(A.24), (A.25), (A.26)] in Lemma 2 to be $[f_1(a_1, a_2, \lambda; \tau_y) = 0, f_2(a_1, a_2, \lambda; \tau_y) = 0, f_3(a_1, a_2, \lambda; \tau_y) = 0]$. We apply the implicit function theorem to the system to obtain

$$
\begin{bmatrix}
\frac{da_1}{d\tau_y} \\
\frac{da_2}{d\tau_y} \\
\frac{d\lambda}{d\tau_y}
\end{bmatrix} = -
\begin{bmatrix}
\frac{\partial f_1}{\partial a_1} & \frac{\partial f_1}{\partial a_2} & \frac{\partial f_1}{\partial \lambda} \\
\frac{\partial f_2}{\partial a_1} & \frac{\partial f_2}{\partial a_2} & \frac{\partial f_2}{\partial \lambda} \\
\frac{\partial f_3}{\partial a_1} & \frac{\partial f_3}{\partial a_2} & \frac{\partial f_3}{\partial \lambda}
\end{bmatrix}^{-1}
\begin{bmatrix}
\frac{\partial f_1}{\partial \tau_y} \\
\frac{\partial f_2}{\partial \tau_y} \\
\frac{\partial f_3}{\partial \tau_y}
\end{bmatrix}.
$$

(A.27)

It follows from (A.27) that

$$
\frac{da_1}{d\tau_y} > 0 \text{ and } \frac{da_2}{d\tau_y} < 0.
$$

(A.28)

Starting from $\tau_y = 0$, which is the no-disclosure case studied in Propositions 1 and 2, increasing disclosure quality increases $a_1^*$ and decreases $a_2^*$. Let $\tau_y^*$ be the precision level such that $a_2^*$ is lowered to zero for the first time.

For $\tau_y > \tau_y^*$, one can show that $a_2^*$ will stay at zero. Substituting $a_2^* = 0$ back to Lemma 2 characterizes the equilibrium pricing function. In particular, the equilibrium is characterize by the first-order condition $k_1 = a_1 - (\alpha_0 + \alpha_y) = 0$ and the investors’ indifferent condition $k_2 = \exp(\rho * F) - \sqrt{1 + \frac{\tau_0}{\tau_y + (\lambda r e / \rho)^2 r_e + 1 / \Sigma}} = 0$. We can then derive

$$
\begin{bmatrix}
\frac{da_1}{d\tau_y} \\
\frac{d\lambda}{d\tau_y}
\end{bmatrix} = -
\begin{bmatrix}
\frac{\partial k_1}{\partial a_1} & \frac{\partial k_1}{\partial \lambda} \\
\frac{\partial k_2}{\partial a_1} & \frac{\partial k_2}{\partial \lambda}
\end{bmatrix}^{-1}
\begin{bmatrix}
\frac{\partial k_1}{\partial \tau_y} \\
\frac{\partial k_2}{\partial \tau_y}
\end{bmatrix},
$$

(A.29)
and show, based on (A.29), for \( \tau_y > \tau_y^* \):

\[
\frac{da_1}{d\tau_y} < 0 \quad \text{and} \quad \frac{d\lambda}{d\tau_y} < 0. \tag{A.30}
\]

Collecting conditions completes the proof. ■

**Proof of Corollary 3.** We know from Proposition 3 that investors’ welfare \( E[U] = -(\sqrt{1 + \frac{\text{var}[E(v|p) - p]}{\text{var}(v|p)}})^{-1} \) has a one-to-one increasing relation to ratio \( R = \frac{\text{var}[E(v|p) - p]}{\text{var}(v|p)} \). Using the pricing function derived in Lemma 2, we can show

\[
R = \frac{(\tau_\theta + \tau_y) (\lambda^2 \tau_e^2 + \tau_e + \tau_\theta + \tau_y)}{r^2 \tau_e \tau_e (\lambda \tau_e (\lambda r^2 \tau_e + 1) + \tau_\theta + \tau_y)^2}, \tag{A.31}
\]

where \( r = 1/\rho \) and \( \tau_\theta = 1/(\Sigma - a_2) \). We apply chain rule to obtain

\[
\frac{dR}{d\tau_y} = \frac{\partial R}{\partial \tau_y} + \frac{\partial R}{\partial \lambda} \frac{d\lambda}{d\tau_y} + \frac{\partial R}{\partial \tau_\theta} \frac{da_2}{d\tau_y}. \tag{A.32}
\]

We can verify that \( \frac{dR}{d\tau_y} > 0 \) after substituting \( \frac{d\lambda}{d\tau_\theta} \) and \( \frac{da_2}{d\tau_y} \) derived from the implicit function theorem (A.27), ■

42